Thyristor Controlled Series Compensator

UNIT 3
Structure

- 3.1 Series compensation
- 3.2 Advantages of TCSC
- 3.3 TCSC Controllers
- 3.4 Operation of TCSC
- 3.5 Modelling of TCSC
SERIES COMPENSATION

3.1.1 Fixed-Series Compensation

- Series capacitors offer certain major advantages over their shunt counterparts:
  - The reactive power increases as the square of line current.
  - For achieving the same system benefits as those of series capacitors, shunt capacitors that are three to six times more reactive power–rated than series capacitors.
SERIES COMPENSATION

- Furthermore, shunt capacitors typically must be connected at the line midpoint, whereas no such requirement exists for series capacitors. Expanded functionality of products.

- Let $Q_{se}$ and $Q_{sh}$ be the ratings of a series and shunt capacitor, respectively, to achieve the same level of power transfer through a line that has a maximum angular difference of $\delta_{max}$ across its two ends:

$$\frac{Q_{se}}{Q_{sh}} = \tan^2\left(2 \delta_{max}\right)$$
3.1.2 The Need for Variable-Series Compensation

1. enhanced base-power flow and loadability of the series-compensated line;
2. additional losses in the compensated line from the enhanced power flow; And
3. increased responsiveness of power flow in the series-compensated line from the outage of other lines in the system.
3.2 Advantages of the TCSC

1. Rapid, continuous control of the transmission-line series-compensation level.

2. Dynamic control of power flow in selected transmission lines within the network to enable optimal power-flow conditions and prevent the loop flow of power.

3. Damping of the power swings from local and inter-area oscillations.
3.2 Advantages of the TCSC

4. Suppression of subsynchronous oscillations.

5. Enhanced level of protection for series capacitors. A fast bypass of the series capacitors can be achieved through thyristor control when large overvoltages develop across capacitors following faults. Likewise, the capacitors can be quickly reinserted by thyristor action after fault clearing to aid in system stabilization.
3.2 Advantages of the TCSC

7. Voltage support.
3.3 THE TCSC CONTROLLER

Basic Module
3.3 THE TCSC CONTROLLER
Fig. 1.17. TCSC (Source: ABB)
Figure 7.2  A typical TCSC system.
3.4 OPERATION OF THE TCSC

A simple understanding of TCSC functioning can be obtained by analyzing the behavior of a variable inductor connected in parallel with an FC, as shown in Fig. 7.3. The equivalent impedance, $Z_{eq}$, of this $LC$ combination is expressed as

$$Z_{eq} = \left( j \frac{1}{\omega C} \right) (j\omega L) = j \frac{1}{\omega C} \frac{1}{\omega L}$$

(7.2)
3.4 OPERATION OF THE TCSC

If \( \omega \dot{C} = (1/\omega L) > 0 \) or, in other words, \( \omega L > (1/\omega C) \),

the reactance of the FC is less than that of the parallel-connected variable reactor and that this combination provides a variable-capacitive reactance.

\[
\begin{align*}
\alpha &= 18^\circ \quad X_L = \tan \alpha \\
X_{TCSC} &= -j X_C \\
X_{TCSC} &= X_C \text{ at } d = 90^\circ
\end{align*}
\]
Modes of Operation

1. Bypassed Thyristor mode

- The thyristors are made to fully conduct with a conduction angle of 180°.
- The TCSC module behaves like a parallel capacitor-inductor combination.
- However, the net current through the module is inductive, for the susceptance of the reactor is chosen to be greater than that of the capacitor.
- Also known as the thyristor-switched-reactor (TSR) mode.
2. The blocked-thyristor mode

- also known as the waiting mode,
- the firing pulses to the thyristor valves are blocked.
- If the thyristors are conducting and a blocking command is given, the thyristors turn off as soon as the current through them reaches a zero crossing.
- The TCSC module is thus reduced to a fixed-series capacitor, and the net TCSC reactance is capacitive.
3. The partially conducting thyristor (capacitive-vernier)

- This mode allows the TCSC to behave either as a continuously controllable capacitive reactance or as a continuously controllable inductive reactance.

- A variant of this mode is the capacitive-vernier-control mode, in which the thyristors are fired when the capacitor voltage and capacitor current have opposite polarity.

To preclude resonance, the firing angle $\alpha$ of the forward-facing thyristor, as measured from the positive reaching a zero crossing of the capacitor voltage, is constrained in the range $\alpha_{min} \leq \alpha \leq 180^\circ$. 
4. The partially conducting thyristor (inductive-vernier) mode.

Another variant is the inductive-vernier mode, in which the TCSC can be operated by having a high level of thyristor conduction. In this mode, the direction of the circulating current is reversed and the controller presents a net inductive impedance.
Based on three modes of operation

- 1. *Thyristor-switched series capacitor (TSSC)*, which permits a discrete control of the capacitive reactance.

- 2. *Thyristor-controlled series capacitor (TCSC)*, which offers a continuous control of capacitive or inductive reactance. (The TSSC, however, is more commonly employed.)
Analysis

\[ V_c(t) = V_c(t) \]

\[ i_s(t) = i_c(t) + i_T(t) \]

\[ i_T(t) = i_s(t) - i_c(t) \]

\[ V_c(t) = V_c(t) \]

\[ \frac{d \hat{I}_c(t)}{dt} = \frac{d \hat{I}_c(t)}{dt} \]

\[ i_c(t) = \frac{C}{L} \frac{d V_c(t)}{dt} \]

\[ i_T(t) = \frac{C}{L} \frac{d^2 i_T(t)}{dt^2} \]

\[ i_T(t) = i_s(t) - LC \frac{d^2 i_T(t)}{dt^2} \]

\[ -LC \frac{d^2 i_T(t)}{dt^2} + i_T(t) = i_s(t) \]

\[ \left( D^2 + \frac{1}{LC} \right) i_T(t) = \frac{i_s(t)}{LC} \]

\[ D = \pm \frac{1}{V_c(t)} \text{purely imaginary} \]

\[ e \Phi = A \cos \frac{t}{\sqrt{LC}} + B \sin \frac{t}{\sqrt{LC}} \]

\[ \text{P.I.} = \left( \frac{\text{Im} \cos (\omega t)}{LC} \right) = \frac{\text{Im} \cos (\omega t) / LC}{-W^2 + W^2} \]
\[ P_2 = \frac{(\text{Im} \cos(\omega t))}{\omega^2 - \omega_0^2} \]

\[ = \frac{\omega_0^2}{\omega^2} \text{Im} \cos(\omega t) \]

\[ i_T(t) = A \cos \frac{t}{\sqrt{LC}} + B \sin \frac{t}{\sqrt{LC}} + \frac{\omega_r^2}{\omega_0^2 - \omega^2} \text{Im} \cos(\omega t) - \text{(i)} \]

\[ i_T(0) = 0, \quad t = \frac{\beta}{\omega} \]

\[ I^* = \frac{\omega^2 \beta}{\omega^2} \]

\[ i_T(0) = 0, \quad \beta = -\omega t, \quad t = -\frac{\beta}{\omega} \]

\[ \text{Condition (ii)} \]

\[ i_T(t) = A \cos \frac{t}{\sqrt{LC}} + B \sin \frac{t}{\sqrt{LC}} + \text{Im} \cos(\omega t) \frac{\omega_r^2}{\omega_0^2 - \omega^2} \]

\[ \theta = A \cos \omega_0 \left( \frac{\beta}{\omega} \right) + B \sin \omega_r \left( \frac{\beta}{\omega} \right) + \text{Im} \cos \omega \left( \frac{\beta}{\omega} \right) \frac{\omega_r^2}{\omega_0^2 - \omega^2} \]

\[ \theta = A \cos \left( \frac{\omega_r}{\omega} \right) \beta + B \sin \left( \frac{\omega_r}{\omega} \right) \beta + \text{Im} \cos \beta \frac{\omega_r^2}{\omega_0^2 - \omega^2} \]

\[ \text{Condition (ii)} \]

\[ i_T(0) = A \cos \omega_0 \left( -\frac{\beta}{\omega} \right) + B \sin \omega_r \left( -\frac{\beta}{\omega} \right) + \text{Im} \cos \omega \left( -\frac{\beta}{\omega} \right) \frac{\omega_r^2}{\omega_0^2 - \omega^2} \]

\[ \theta = A \cos \left( \frac{\omega_r}{\omega} \right) \beta - B \sin \left( \frac{\omega_r}{\omega} \right) \beta + \text{Im} \cos \beta \frac{\omega_r^2}{\omega_0^2 - \omega^2} \]

Adding (1) + (2), we get:

\[ 2 A \cos \left( \frac{\omega_r}{\omega} \beta \right) + 2 \text{Im} \cos \beta \frac{\omega_r^2}{\omega_0^2 - \omega^2} = 0 \]
\[ A \cos \omega_0 (k_β) = - \frac{1}{2} \Im \cos (\beta) \cos (k_β) \]

\[ A = - \frac{k^2}{k^2 - 1} \frac{\Im \cos (\beta)}{\cos (k_β)} \]

Subtracting (1) \& (2)

\[ 2 \beta \sin \left( \frac{\omega x}{\omega} \right) \beta \to \infty \]

\[ \boxed{\beta = \frac{\omega x}{\omega} \beta} \]

Substituting \( A \) and \( \beta \) in (1) \& (2), we get:

\[ \ell_T(t) = \frac{k^2}{k^2 - 1} \frac{\Im}{\cos (k_β)} \left[ \cos (\omega t) - \cos \left( \frac{\omega (k x)}{\omega} \right) \right] \]

\[ \ell_T(t) = \frac{k^2}{k^2 - 1} \frac{\Im}{\cos (k_β)} \left[ \sin (\omega t) + \frac{\cos \beta (\omega - \sin (\omega t))}{\cos (k_β)} \right] \]

Put \( t = \frac{-\beta}{\omega} \)

\[ \ell_T(t) = L \frac{d}{dt} \left( \frac{k^2}{k^2 - 1} \frac{\Im}{\cos (k_β)} \left[ \sin (\omega t) + \frac{\cos \beta (\omega - \sin (\omega t))}{\cos (k_β)} \right] \right) \]

\[ = L \frac{k^2}{k^2 - 1} \frac{\Im}{\cos (k_β)} \left[ \sin (\omega t) + \frac{\cos \beta (\omega - \sin (\omega t))}{\cos (k_β)} \right] \]

\[ = L \frac{k^2}{k^2 - 1} \frac{\Im}{\cos (k_β)} \left[ \sin (\omega t) + \frac{\cos \beta (\omega - \sin (\omega t))}{\cos (k_β)} \right] \]
\[ \psi = L \cdot \frac{k^2}{k^2 - 1} \text{Im} \left( \frac{w_r}{w} \left( -\cos(\beta) \tan(k \beta) + \frac{\sin(\beta)}{k} \right) \right) \]

\[ = 1 \cdot \frac{k^2}{k^2 - 1} \text{Im} \left( \frac{w_r}{w} \left( \cos(\beta) \tan(k \beta) + \frac{\sin(\beta)}{k} \right) \right) \]

\[ = \frac{\text{Im} \left( \frac{w_r}{w} \right)}{k^2 - 1} \left( k \cos(\beta) \tan(k \beta) + \frac{\sin(\beta)}{k} \right) \]

\[ V_{c1} = \frac{\text{Im} \left( \frac{w_r}{w} \right)}{k^2 - 1} \left( k \cos(\beta) \tan(k \beta) + \frac{\sin(\beta)}{k} \right) \]

\[ V_c(t) = \begin{cases} \frac{L d i(t)}{dt} & (-\beta \leq wt \leq \beta) \\ V_{c2} + \frac{1}{C} \int \text{Im} \cos(wt) \, dt & (\beta \leq wt \leq \pi - \beta) \end{cases} \]

Substitute \( wt = -\beta \), in (3), we get:

\[ V_{c1} = \frac{\text{Im} \left( \frac{w_r}{w} \right)}{k^2 - 1} \left( -k \cos(\beta) \right) \]

\[ V_{c1} = -V_{c2} \]

\[ V_{c2} = \frac{-\text{Im} \left( \frac{w_r}{w} \right)}{k^2 - 1} \left[ \sin(\beta) - k \cos(\beta) \tan(k \beta) \right] \]

**vget**: Voltage across the capacitor at fundamental

\[ V_{cF} = \text{Fundamental voltage across the capacitor} \]

\[ a_1 = 2 \]

\[ b_1 = \frac{2}{T_0} \int_0^T f(t) \sin(wt) \, dw \]

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voltage across the capacitor at fundamental

\[ V_{cf} = \text{Fundamental voltage across capacitor} \]

\[ a_1 = 0 \]

\[ b_1 = \frac{2}{T} \int_0^T f(t) \sin(\omega t) \, dt \]

\[ b_1 = \frac{2\pi y}{2\pi} \int_0^{\pi/2} V_c(t) \sin(\omega t) \, dt \]

\[ \omega = \frac{V_c}{k^2 - 1} \]

\[ \text{Im} \left[ k \cos \beta \sin(\omega t) - \sin(\omega t) \right] \]

\[ \sin(\omega t) \, dt \]

\[ V_{e2} = \frac{1}{c} \int \frac{\sin \omega t}{\omega} \, dt \]

\[ V_{e2} = V_{e2} + \frac{1}{c} \int \frac{\sin \omega t}{\omega} \, dt \]

\[ = \frac{\text{Im} \left[ k \cos \beta \sin \omega t - \alpha \sin \omega t \right]}{k^2 - 1} \]

\[ + \frac{\text{Im} \left[ \sin(\omega t) \right]}{\omega} \]

\[ + \frac{\text{Im} \left[ \sin(\omega t) - \sin \beta \right]}{\omega} \]
Integrating \( V_{CF} \),

\[
V_{CF} = \int V_{CF1} + \int V_{CF2}
\]

\[
V_{CF1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \left[ \frac{k \cos \beta}{\cos \omega \text{kt}} \sin(\omega \text{kt}) \sin(\omega t) \right] \sin(\omega t) \, d(\omega t)
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \frac{k \cos \beta}{\cos \omega \text{kt}} \sin(\omega \text{kt}) \sin(\omega t) \sin^2(\omega t) \, d(\omega t)
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \frac{k \cos \beta}{\cos \omega \text{kt}} \frac{\cos(\omega \text{kt} - \omega t) + \cos(\omega \text{kt} + \omega t)}{2} \, d(\omega t)
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \frac{k \cos \beta}{2 \cos \omega \text{kt}} \left[ \frac{\cos(\omega \text{kt} - \omega t) - \cos(\omega \text{kt} + \omega t)}{2} \right] \, d(\omega t)
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \frac{k \cos \beta}{2 \cos \omega \text{kt}} \left[ \frac{\sin(\omega \text{kt} - \omega t)}{\omega (k-1)} - \frac{\sin(\omega \text{kt} + \omega t)}{\omega (k+1)} \right] \, d(\omega t)
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \frac{k \cos \beta}{2 \cos \omega \text{kt}} \left[ \frac{\sin(\omega \text{kt} - \omega t)}{\omega (k-1)} - \frac{\sin(\omega \text{kt} + \omega t)}{\omega (k+1)} \right] \, d(\omega t)
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{\text{Im} X_c}{k^2-1} \frac{k \cos \beta}{2 \cos \omega \text{kt}} \left[ \frac{\sin(\omega \text{kt} - \omega t)}{\omega (k-1)} - \frac{\sin(\omega \text{kt} + \omega t)}{\omega (k+1)} \right] \, d(\omega t)
\]
\[ \begin{align*}
&= \frac{4}{\pi} \Im \chi_c \left[ \frac{K \cos \beta}{2 \cos k \beta} \right] \frac{(K+1) \sin (\beta k - \beta) - (K-1) \sin (\beta k + \beta)}{(k^2 - 1)} \\
&= \frac{4}{\pi} \Im \chi_c \left[ \frac{A - B}{k^2 - 1} \right] \quad \cdots (\text{C})
\end{align*} \]

\[ \beta = \frac{1}{2} \left( \beta - \frac{2 \sin \beta}{\beta} \right) \]
\[ = \frac{\beta}{2} - 2 \sin \beta \cos \beta \quad \frac{\beta}{2} \]
\[ = \frac{\beta}{2} - \frac{\cos^2 \beta + \tan \beta}{2} \]

\[ \beta = \frac{K \cos \beta}{2 \cos k \beta} \left[ \frac{(K+1) \sin (\beta k - \beta) - (K-1) \sin (\beta k + \beta)}{k^2 - 1} \right] \]
\[ = \frac{K \cos \beta}{2 \cos k \beta} \left[ \frac{K \cos (\beta k) \cos (\beta) - \cos (\beta k) \sin (\beta)}{k^2 - 1} \right] - (K-1) \left[ \sin (\beta k) \cos (\beta) + \cos (\beta k) \sin (\beta) \right] \]
\[ = \frac{K \cos \beta}{2 \cos k \beta} \left[ -2K \cos (\beta k) \sin (\beta) + 2 \sin (\beta k) \cos (\beta) \right] \quad \frac{K \cos \beta}{2 \cos k \beta} \left[ -2K \cos (\beta k) \sin (\beta) + \sin (\beta k) \cos (\beta) \right] \]
\[ = -K^2 \cos \left( \frac{\beta k}{\cos k} \right) \cos (\beta) \sin (\beta) + K \sin (\beta k) \cos^2 (\beta) \]
\[ \cos (k \beta) \quad (k^2 - 1) \]
\[ V_{E1} = \frac{4}{\pi} \text{Im} \frac{Xc}{B^2 - 1} \left[ \frac{-k^2 \cos^2(\beta) \tan(\beta)}{k^2 - 1} + i \frac{k \tan(\beta) \cos^2(\beta)}{k^2 - 1} \right] \]

Substituting (1) and (3) in (4) we get:

\[ V_{E1} = \frac{4}{\pi} \text{Im} \frac{Xc}{B^2 - 1} \left[ \frac{-k^2 \cos^2(\beta) \tan(\beta)}{k^2 - 1} + i \frac{k \tan(\beta) \cos^2(\beta)}{k^2 - 1} \right] \]

\[ V_{E2} = \frac{4}{\pi} \int \left[ \frac{V_c(b) \sin(\omega t)}{B} \right] d(\omega t) \]

\[ V_c(b) = V_{c2} + \frac{\text{Im} \frac{Xc}{B^2 - 1}}{\omega c} \left[ \sin(\omega t) - \sin(\beta) \right] \]

\[ = \frac{4}{\pi} \int \left[ V_{c2} + \frac{\text{Im} \frac{Xc}{B^2 - 1}}{\omega c} \left[ \sin(\omega t) - \sin(\beta) \right] \right] \sin(\omega t) d(\omega t) \]

(1) \Rightarrow

\[ \frac{4}{\pi} \int \left[ V_{c2} \cdot \sin(\omega t) \right] d(\omega t) \]

\[ = \frac{4}{\pi} V_{c2} \left[ \cos(\omega t) \right] \]

\[ = \frac{4}{\pi} V_{c2} \cos \beta \]

\[ = \frac{4}{\pi} \text{Im} \frac{Xc}{k^2 - 1} \left[ \frac{K \sin(K\beta) \cos(\beta)}{\cos(K\beta)} - \sin \beta \right] \cos \beta \]

(1) \Rightarrow

\[ \frac{4}{\pi} \text{Im} \frac{Xc}{k^2 - 1} \left[ \frac{K \tan(K\beta) \cos^2(\beta)}{k^2 - 1} - \frac{2 \sin \beta}{2} \right] \]
\( (3) \Rightarrow \)

\[
\frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \frac{Im\left\{ \frac{1}{\omega_c} \left[ \sin(\omega_c t) - \sin(\beta t) \right] \sin(\omega t) \right\}}{\beta} dt
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left[ \sin^2(\omega t) \right] \sin(\beta t) dt
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left[ \frac{1}{2} \left( 1 - \cos(2\omega t) \right) \right] \sin(\beta t) dt
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left[ \frac{1}{2} \left( \frac{1}{2} - \sin^{2}(\omega t) \right) + \sin(\beta t) \left( \cos(\omega t) \right) \right] dt
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left( \frac{1}{2} \left( \frac{1}{2} - \beta - \frac{\sin(2\beta t)}{2} \right) + \sin(\beta t) \left( \cos(\omega t) \right) \right) dt
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left( \frac{1}{2} \left( \frac{1}{2} - \beta - \frac{\sin(2\beta t)}{2} \right) \right) dt
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left( \frac{1}{2} \left( \frac{1}{2} - \beta - \frac{\sin(2\beta t)}{2} \right) \right) dt
\]

\[
V_{CF_2} = 0 + V_{CF_1}
\]

\[
= \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left[ k \tan(\omega t) \cos^{2}(\beta t) - \frac{\sin(2\beta t)}{2} \right] dt
\]

\[
+ \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} \left[ \frac{\sin(2\beta t)}{2} \right] dt
\]

\[
V_{CF} = V_{CF_1} + V_{CF_2}
\]
\[ V_C = \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \left\{ -\frac{k^2}{k^2-1} \left[ \cos^2(\beta) \tan(\beta) \right] + \frac{\text{Im} (k^2 \beta) \cos^2(\beta)}{k^2-1} \right\} - \frac{1}{2} + \frac{\cos^2(\beta) \tan(\beta)}{2} \]

\[ + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \left[ \frac{\text{Im}(k \beta) \cos^2(\beta) - \sin 2 \beta}{2} \right] \]

\[ + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \left[ \frac{\pi}{4} - \frac{\beta}{2} - \frac{\sin 2 \beta}{4} \right] \]

\[ = -\frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \left[ \frac{k^2 \cos^2(\beta) \tan(\beta)}{k^2-1} + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \frac{\text{Im}(k \beta) \cos^2(\beta)}{k^2-1} \right] \]

\[ = \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \cos^2(\beta) \tan(\beta) \left[ \frac{1}{2} - \frac{k^2}{k^2-1} \right] \]

\[ + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \pi \tan(\beta) \cos^2(\beta) \left[ \frac{1}{k^2-1} + 1 \right] \]

\[ + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \sin^2(\beta) \left[ \frac{1}{2(k^2-1)} - \frac{1}{4} \right] \]

\[ + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \left[ \frac{\pi}{4} - \frac{\beta}{2} - \frac{1}{k^2-1} \right] \]

\[ + \frac{4}{\pi} \frac{\text{Im} X_C}{k^2-1} \frac{1}{2} \left[ \frac{1}{k^2-1} + 1 \right] \]
\[ V_C = \frac{4}{n} \frac{\text{Im} \, X_C}{k^2 - 1} \cos^2 \beta \tan \beta \left[ \frac{k^2 - 1 - 2k^2}{2(k^2 - 1)} \right] \\
+ \frac{4}{n} \frac{\text{Im} \, X_C}{k^2 - 1} k \tan (k^2 \beta) \cos^2 \beta \left[ \frac{1 + k^2}{k^2 - 1} \right] \\
- \frac{4}{n} \frac{\text{Im} \, X_C}{k^2 - 1} \left[ \frac{1 + k^2}{k^2 - 1} \right] + \text{Im} \, X_C \\
- \frac{2}{n} \frac{\text{Im} \, X_C}{k^2 - 1} \sin 2\beta \left[ \frac{2 - \beta}{4(k^2 - 1)} \right] \\
+ \frac{4}{n} \frac{\text{Im} \, X_C}{k^2 - 1} \frac{k^2}{k^2 - 1} \left[ \frac{k^2}{2(k^2 - 1)} \right] \\
+ \frac{4}{n} \frac{\text{Im} \, X_C}{k^2 - 1} \left[ \frac{1 - k^2}{4(k^2 - 1)} \right] + \text{Im} \, X_C \\
\]

\[ k_{\text{fisc}} = \left( \frac{V_{CE}}{\text{Im}} \right) = \frac{X_C - X_C^2}{X_C - X_C} \left[ \frac{2\beta + \sin 2\beta}{n} \right] \\
\]

\[ \frac{4 \frac{X_C^2}{X_C - X_C}}{k^2 - 1} \left( \frac{k \tan (k^2 \beta) - \tan (\beta)}{n} \right) \]
Conclusion

The TSSC offers the following benefits compared to mechanically switched series capacitors:

1. The thyristor switches allow an unlimited number of operations without any wear. This capability is used to alter the degree of line compensation more frequently and to achieve a greater control over the power flow.
2. Exact switching instants (point-of-voltage waveforms) can be selected with thyristors, which significantly minimizes the switching transients. In contrast, the switching of mechanical breakers is unsynchronized.
3. A very rapid speed of response, in which the time between the initiation of a control signal and a capacitor insertion, or bypass, is typically less than a half-cycle (8 ms for 60 Hz). Thus, in case a major tie-line suffers an outage, the power-transfer capability of an alternative line can be increased rapidly through the TSSC.

4. No generation of harmonics.