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Question Paper Code : 91585

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Common to B.Tech. Information Technology)

(Regulation: 2008/2010)

(Common to PTMA 2265/10144 CS 501 — Discrete Mathematics for
B.E. (Part -Time) Third Semester — Computer Science and Engineering —
Regulation 2009/2010)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.
2. Find a counter example, if possible, to these universally quantified statements, whose the universe of discourse for all variables consists of all integers.
 - (a) $\forall x \forall y (x^2 = y^2 \rightarrow x = y)$.
 - (b) $\forall x \forall y (xy \geq x)$.
3. How many permutations of $\{a, b, c, d, e, f, g\}$ and with a ?
4. In how many ways can a $2 \times n$ rectangular board be tiled using 1×2 and 2×2 pieces?
5. Define isomorphism of directed graphs.
6. What do you strongly connected components of a telephone call graph represent?
7. Give an example for homomorphism.
8. Define semigroups and Monoids.

9. What values of the Boolean variables x and y satisfy $xy = x + y$?
10. Define a Lattice. Give suitable example.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Establish this logical equivalences, where A is a proposition not involving any quantifiers. Show that $(\forall x p(x)) \wedge A = \forall x (p(x) \wedge A)$ and $(\exists x p(x)) \wedge A = \exists x (p(x) \wedge A)$. (8)
- (ii) Show that $\exists x p(x) \wedge \exists x Q(x)$ and $\exists x (p(x) \wedge Q(x))$ are not logically equivalent. (8)

Or

- (b) (i) Use quantifiers and predicates to express the fact that $\lim_{x \rightarrow a} f(x)$ does not exist. (8)
- (ii) Show that $\forall x p(x) \wedge \exists x Q(x)$ is equivalent to $\forall x \exists y (p(x) \wedge Q(y))$. (8)
12. (a) (i) Show that if n and k are positive integers then $\binom{n+1}{k} = (n+1) \binom{n}{k-1} / k$. Use this identity to construct an inductive definition of the binomial co-efficients. (8)
- (ii) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5, a_1 = -9$, and $a_2 = 15$. (8)

Or

- (b) (i) Use the principle of inclusion - exclusion to derive a formula for $\phi(n)$ when the prime factorization of n is $n = p_1^{a_1} p_2^{a_2} \dots p_m^{a_m}$. (8)
- (ii) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, with the initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$. (8)
13. (a) (i) Describe a discrete structure based on a graph that can be used to model airline routes and their flight times. (8)
- (ii) Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges. (8)

Or

- (b) (i) Show that isomorphism of simple graphs is an equivalence relation. (8)
- (ii) Derive an algorithm for constructing Euler path in directed graphs. (8)
14. (a) (i) Show that the intersection of any two congruence relations on a set is also a congruence relation. (8)
- (ii) Show that a semigroup with more than one idempotent cannot be a group. Give an example of a semigroup which is not group. (8)

Or

- (b) (i) Let $\langle H_1, * \rangle$ and $\langle H_2, * \rangle$ be subgroups of a group $\langle G, * \rangle$. Show that $\langle H_1 \cap H_2, * \rangle$ is also a subgroup of $\langle G, * \rangle$. Also show that, in general $\langle H_1 \cup H_2, * \rangle$ is not a subgroup of $\langle G, * \rangle$ except when $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$. (8)
- (ii) Discuss Ring and Fields with suitable examples. (8)
15. (a) (i) Show that a complemented, distributive lattice is a Boolean algebra. (8)
- (ii) Show that the De Morgan's laws hold in a Boolean algebra. That is, show that for all x and y , $\overline{(x \vee y)} = \overline{x} \wedge \overline{y}$ and $\overline{(x \wedge y)} = \overline{x} \vee \overline{y}$. (8)

Or

- (b) (i) Show that every non - empty subset of a lattice has a least upper bound and a greatest lower bound. (8)
- (ii) Show that every totally ordered set is a lattice. (8)