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07MCA11B

First Semester MCA Degree Examination, May/June 2010

Mathematics

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

1 a. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then prove that $(m^2 - n^2)^2 = 16mn$. (06 Marks)

b. If $A = 45^\circ$, verify that $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$. (07 Marks)

c. With usual notations, prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$. (07 Marks)

2 a. If $\sin(A + iB) = x + iy$, prove that $x^2 \operatorname{cosec}^2 A - y^2 \sec^2 A = 1$. (06 Marks)

b. Using De Moivre's theorem, solve the equation $x^5 + 1 = 0$. (07 Marks)

c. Prove that $(1 + \cos \theta - i \sin \theta)^n + (1 + \cos \theta + i \sin \theta)^n = 2^{n+1} \cos^n \left(\frac{\theta}{2} \right) \cos \left(\frac{n\theta}{2} \right)$. (07 Marks)

3 a. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{bmatrix}$,
verify that $A(B+C) = AB+AC$. (06 Marks)

b. Determine the roots of the matrix $\begin{bmatrix} 3 & 5 & 7 & 8 & 4 \\ -1 & 2 & 3 & 1 & 3 \\ 4 & 5 & 1 & 2 & -1 \\ 1 & -5 & 6 & -7 & -8 \end{bmatrix}$. (07 Marks)

c. Express the matrix A as the sum of symmetric and skew-symmetric matrices

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix} \quad (07 \text{ Marks})$$

4 a. Test the consistency and solve the system $x + y + z = 6$, $x - y + 2z = 5$ and $3x + y + z = 8$. (06 Marks)

b. Find the eigen value and the eigen vector corresponding to the least eigen value of the

matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (07 Marks)

c. Using the Cayley-Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8=50, will be treated as malpractice.

5 a. Find the nth derivative of $\frac{x}{6x^2 - x - 2}$. (06 Marks)

b. If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y = 0$. (07 Marks)

c. Find the nth derivative of $e^{ax} \cos(bx + c)$. (07 Marks)

6 a. Find the angle of intersection between two curves

$$r = a \sec^2 \frac{\theta}{2} \quad \text{and} \quad r = b \operatorname{cosec}^2 \frac{\theta}{2}. \quad (06 \text{ Marks})$$

b. Find the pedal equation of the curve $l/r = 1 + e \cos \theta$. (07 Marks)

c. Evaluate (i) $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$ (ii) $\lim_{x \rightarrow \pi/2} (1 - \sin x) \tan x$ (07 Marks)

7 a. Evaluate (i) $\int x \log x \, dx$ (ii) $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} \, dx$ (06 Marks)

b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} \, dx$. (07 Marks)

c. Evaluate $\int_0^\pi \frac{dx}{5 + 4 \cos x}$. (07 Marks)

8 a. Solve $\left(\frac{2xy+1}{y}\right)dx + \frac{y-x}{y^2}dy = 0$. (06 Marks)

b. Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$. (07 Marks)

c. Solve $(x + 2y^3)\frac{dy}{dx} = y$. (07 Marks)
