Code: 9ABS302

B.TECH II Year I Semester (R09) Regular & Supplementary Examinations November 2012

MATHEMATICS-III

(Common to Electrical & Electronics Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Communication Engineering and Electronics & Computer Engineering)

Time: 3 hours | Max. Marks: 70

Answer any FIVE questions
All questions carry equal marks

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1. (a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
   (b) Find the value of $\Gamma\left(1/2\right)$ and hence evaluate $\int_0^\infty e^{-x^2}dx$ using Γ funcition.
   (c) Prove that $P_{n+1}^1 + P_n^1 = P_0 + 3P_1 + 5P_2 \pm ---- + (2n+1)P_n$.

2. (a) Define analyticity of a complex function at a point P and in a domain D. Prove that the real and imaginary parts of an analytic function satisfies C-R equations.
   (b) Show that the function defined by $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$ at $z \neq 0$ and $f(0) = 0$ is continous and satisfies C-R equations at the origin but $f^1(0)$ does not exist.

3. (a) Find all values of z which satisfy $\sin z = 2$.
   (b) Find all principal values of $(1 + i\sqrt{3})^{(1+i\sqrt{3})}$.

4. (a) Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ if C is the square with vertices at $1 \pm i$ and $-1 \pm i$.
   (b) Let c be the circle $z = \exp(i\theta)$described from $\theta = -\pi$ to π and K is any real constant. Show that $\int_c \frac{e^{kz}}{z}dz = 2\pi i$ then write the integral in terms of θ to derive the formula $\int_0^\pi e^{k\cos\theta}\cos(k\sin\theta)\,d\theta = \pi$.

5. (a) If $f(z)$ is analytic inside and on a simple closed circle if C with center at a, then prove that for z inside C

$$f(z) = f(a) + f^1(a)(z-a) + \frac{f^{11}(a)}{2!}f^1(z-a)^2 + \frac{f^{111}(a)}{3!}(z-a)^3 + \cdots \ldots \ldots$$

   (b) Obtain all the Laurent series of the function $\frac{7z-2}{(z+1)z(z-2)}$ about $z = -1$.

6. (a) Show that $\int_0^\pi \frac{a\,d\theta}{a^2+\sin^2\theta} - \pi/\sqrt{1+a^2},$ a>0.
   (b) Evaluate $\int_{-\pi}^{\pi} \frac{a\cos\theta\,d\theta}{a+\cos\theta}$, a>1 using residue theorem.

7. (a) Suppose $f(z)$ and $g(z)$ are analytic within and on a closed curve C and if $|g(z)| < |f(z)|$ on C then prove that $f(z)$ and $f(z) + g(z)$ both have the same number of zeros inside C.
   (b) If the real number $a > e$, prove, by Rouche's theorem, that the equation $e^z = az^n$ has n roots inside the unit circle.

8. (a) Prove that the transformation $w = \sin z$ maps the families of lines $x =$ constant and $y =$constant into two families of confocal central conies.
   (b) Find the image of the infinite strip between the lines $y = 2$ and $y = 4$ under the transformation $w = \sin z$.

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