

[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1569 C Roll No.....

Unique Paper Code : 235666

Name of the Course : B.Sc. Physical Sciences/Applied Physical Sciences

Name of the Paper : Mechanics and Discrete Mathematics : MAPT-606

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any two parts from each question.
3. All questions are compulsory.
4. Marks are indicated.

1. (a) Two beads of weights w and w' can slide on a smooth circular wire in a vertical plane. They are connected by a light string which subtends an angle 2β at the centre of the circle when the beads are in equilibrium on the upper half of the wire. Prove that the inclination α of the string to the horizontal is given by

$$\tan \alpha = \frac{w - w'}{w + w'} \tan \beta \quad (8)$$

- (b) Two light rings can slide on a rough horizontal rod. The rings are connected by a light inextensible string of length a , to the midpoint of which is attached a weight w . Show the greatest distance between the rings, consistent with equilibrium of the system, is $\frac{\mu a}{\sqrt{1 + \mu^2}}$, where μ is the coefficient of friction between either ring and the rod. (8)

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- (c) Find the mass centre of a cubical box with no lid, the sides and bottom being made of the same thin material. (8)

2. (a) Prove that for a particle moving along a curve's the tangential and normal

Components of a velocity are $(\dot{s}, 0)$ and of acceleration are $\left(\frac{dv}{dt}, \frac{v^2}{\rho}\right)$,

where v is the speed and ρ is the radius of curvature of the curve at the point. (8)

- (b) A gun is mounted on a hill of height h above a level plain. Show that if the resistance of air is neglected, the greatest horizontal range for given muzzle velocity v is obtained by firing at an angle of elevation θ such that

$$\operatorname{cosec}^2\theta = 2\left(1 + \frac{gh}{v^2}\right). \quad (8)$$

- (c) (i) A uniform accelerated automobile passes two telephone poles with velocities 10 and 20 mph, Calculate its velocity when it is halfway between the poles. (4)

- (ii) A particle of mass m moves on a straight line under the influence of a force directed towards the origin O on the line and proportional to the distance from O , the force at unit distance is of magnitude mk^2 . The particle passes O with an initial velocity u . If x is the coordinate at time t and v its velocity at that instant, show that

$$v^2 + k^2x^2 = u^2. \quad (4)$$

3. (a) (i) Define complete graph. Using principle of mathematical induction prove

$$\text{that number of edges in } K_n \text{ is } \frac{n(n-1)}{2}. \quad (4)$$

- (ii) Draw the graph K_6 (3)

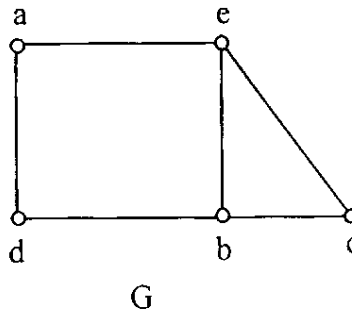
(b) Define in-degree and out-degree of a vertex in a directed graph. Prove that

if $G = (V, E)$ is a directed graph, then $\sum_{x \in V} \text{deg}^-(x) = \sum_{x \in V} \text{deg}^+(x) = |E|$. (7)

(c) Show that if G is a bipartite simple graph with n vertices and e edges,

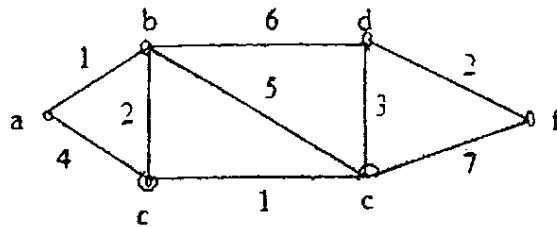
then $e \leq \frac{n^2}{4}$. (7)

4. (a) How many paths of length four are there from a to e in the graph G ? Identify all the paths.



(7)

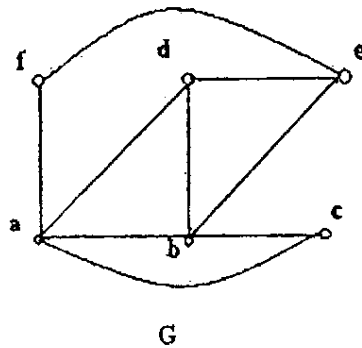
(b) Define weighted graph. Use Dijkstra's algorithm to find the length of a shortest path between a and f in the following weighted graph



(7)

(c) Prove that a tree with n vertices has $n - 1$ edges.

5. (a) Use depth-first search to find a spanning tree for the given graph G (7)



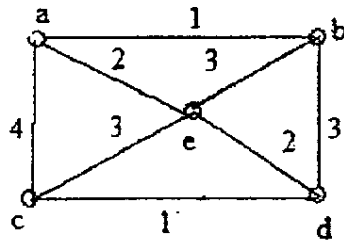
(7½)

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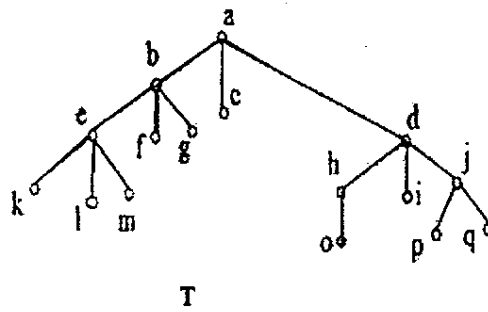
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- (b) Use kruskal's algorithm to find a minimum spanning tree in the following weighted graph



(7½)

- (c) Determine the preorder, postorder traversal of the given rooted tree T.



(7½)

(2500)