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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)
Term-End Examination
June, 2017

00422

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

(Weightage : 70%)

Note : *Question no. **6** is **compulsory**. Attempt any **four** questions from questions no. 1 to 5.*

1. (a) Find a set of generators of A_4. Also find all the possible dimensions of irreducible representations of A_4. *5*

(b) Show that $\mathbf{Q}(4 - i) = \mathbf{Q}(1 + i)$, where $i = \sqrt{-1}$. Further, check whether $\mathbf{Q}(4 - i)$ has an element of degree 4 over $\mathbf{Q}$. *5*

2. (a) Check whether $(\mathbf{R}, +)$ is a free semigroup or not. *3*

(b) Prove that any group of order p^2 is Abelian, where p is a prime. *4*

(c) Is 978-81-7319-269-2 a valid ISBN number ? Give reasons for your answer. *3*

3. (a) Let G be a non-cyclic group of order 21. How many Sylow 3-subgroups does G have ? *3*

(b) Check whether $\mathbf{Q}(2^{1/3}) \mid \mathbf{Q}$ is a normal extension or not. *5*

(c) Give an example, with justification, of a non-regular language. *2*

4. (a) What is a block design ? Further, construct a block design on 4 points with block size 2 and index 3. *5*

(b) Check whether or not

(i) $SP_2(\mathbf{R}) = SL_2(\mathbf{R})$;

(ii) $SP_4(\mathbf{R}) = SL_4(\mathbf{R})$. *5*

5. (a) Let G be a finite group. Can we define a one-one representation of G on a finite-dimensional vector space over $\mathbf{C}$? Justify your answer. *4*

(b) Let G be a group generated by g_1, g_2, g_3, with certain relations $\{r_i \mid i \in I\}$, where I is an indexing set. Let one of the relations be of the form wg_1, where w is a word in g_2 and g_3. Let r_i' be the relation obtained by substituting w^{-1} for g_1 in r_i, and G′ be the group generated by g_2, g_3 with relations $\{r_i' \mid i \in I\}$. Prove that G and G′ are isomorphic. *6*

6. State whether the following statements are *true* or *false*. Give reasons for your answers. *10*

(a) $\phi : \mathbf{Z} \times \mathbf{Q} \to \mathbf{Q} : \phi(z, q) = z - q$ is a group action of $\mathbf{Z}$ on $\mathbf{Q}$.

(b) $\mathbf{Q}(\sqrt{2}) = \mathbf{Q}(\sqrt{3})$.

(c) The conjugacy class of $\sigma \in S_n$, where σ is an even cycle, has an element of signature -1.

(d) Every finitely generated Abelian group is isomorphic to $\mathbf{Z}_n$ for some $n \in \mathbf{N}$.

(e) If $\alpha \in \mathbf{C}$ is algebraic over $\mathbf{Q}$, then $\alpha \in \mathbf{Q}$.