This question paper contains 4+2 printed pages]

Your Roll No.....

3791

B.A. Prog./I **IS**

(D)

OPERATIONAL RESEARCH

Paper I—Foundations of Operational Research

(Admissions of 2004 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory and have equal marks.

Attempt any *two* parts from each Section.

Section I

1. (*a*) Explain the nature of operations/research and its limitations.

(*b*) A ship is to carry 3 types of liquid cargo-X, Y and Z. There are 3,000 litres of X available, 2000 litres of Y available and 1500 litres of Z available. Each litre of X, Y and Z sold fetches a profit of

P.T.O.

Rs. 20, Rs. 35 and Rs. 40 respectively. The ship has 3 cargo holds A, B and C of capacities 2,000, 2,500 and 3,000 litres respectively. From stability consideration, it is required that each hold be filled in the same proportion. Formulate the problem.

(*c*) Define and explain the following terms in an O.K. model :

(*i*) Objective function

(*ii*) Decision variables.

Section II

2. (*a*) Solve the following system of equations :

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + 2x_2 + 3x_3 = 3$$

$$2x_1 + 3x_2 + 4x_3 = 5.$$

(*b*) Express the following matrix as a product of elementary matrices :

$$\begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

(*c*) Show that if λ is an eigen value of A, then $\alpha\lambda$ is that of αA, where α is a scalar with the same eigen vector.

Section III

3. (*a*) Examine whether the set

$$S = \{(x_1, x_2) : x_1^2 + x_2^2 < 1,\ x_1 + x_2 > 1\}$$

is convex set or not.

(*b*) Define :

(*i*) Degenerate solution

(*ii*) Open and closed half space

(*iii*) Extreme point.

(*c*) Define a basic solution to a given system of m simultaneous linear equations in n unknowns. Obtain all the basic solutions to the following system of linear equations :

$$x_1 + 2x_2 + x_3 = 4$$

$$2x_1 + x_2 + 5x_3 = 5.$$

P.T.O.

Section IV

4. (*a*) Find the median of the following distribution.

Wages (in Rs.)	No. of Workers
2000—3000	3
3000—4000	5
4000—5000	20
5000—6000	10
6000—7000	5

(*b*) The first three moments of a distribution about the value 2 are 1, 16 and – 40 respectively. Examine the skewness of the distribution.

(*c*) State and prove Bayes' theorem.

Section V

5. (*a*) Let X be a continuous random variable with p.d.f. given by :

$$f(x) = \begin{cases} kx & 0 \le x < 1 \\ k & 1 \le x < 2 \\ -kx + 3k & 2 \le x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(*i*) Determine the constant k

(*ii*) E[X].

(*b*) Prove that :

(*i*) $E(X) \le E(Y)$, where $P(X \le Y) = 1$

(*ii*) $|E(X)| \le E(|X|)$.

(*c*) Explain, under what conditions and how the binomial distribution can be approximated to the normal distribution.

Section VI

6. (*a*) If X and Y are independent normal variables zero means and standard deviations 9 and 12, respectively and if $X + 2Y$ and $kX - Y$ are uncorrelated, find k.

(*b*) X and Y are discrete random variables. If $Var(X) = Var(Y) = \sigma^2$, $Cov(X, Y) = \sigma^2/2$, find :

(*i*) Var $(2X - 3Y)$

(*ii*) Corr $(2X + 3, 2Y - 3)$.

(*c*) According to the norms established for a reading comprehension test, eighth graders should average 84.3 with a standard deviation of 8.6. If 45 randomly selected eighth graders from a certain school district averaged 87.8, test the null hypothesis $\mu = 84.3$ for that school district against the alternative hypothesis $\mu > 84.3$ using $\alpha = 0.01$.