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(ii) Customers arrive at a sales counter manned by a single person according to a Poisson process with mean rate of 20 per hour. The time required to serve a customer has an exponential distribution with a mean of 100 seconds. Find the average waiting time of a customer (8)

15. (a) Automatic car wash facility operates with only one bay. Cars arrive according to a Poisson process at the rate of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The service time for all cars is constant and equal to 10 minutes. Determine L_s, L_q, W_s and W_q. (16)

Or

(b) Consider a system of two servers where customers from outside the system arrive at sever 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates for server 1 and 2 are 8 and 10 respectively. A customer upon completion of service at server 1 is likely to go to Server 2 or leave the system whereas a departure from server 2 will go 25 percent of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, L_s and W_s. (16)

Reg. No. :

Question Paper Code : 31266

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Computer Science and Engineering

MA 2262 — PROBABILITY AND QUEUEING THEORY

(Common to Information Technology)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = k(1+x)$. Find $P(X < 4)$.

2. Let X be uniformly distributed in (2, 5), find $E(X)$.

3. If the joint pdf of (X,Y) is given by $f(x,y) = 2$, in $0 \le x < y \le 1$, find $E(X)$.

4. State Central limit theorem.

5. Find the variance of the stationary process $\{X(t)\}$, whose auto correlation function is given by $R_{XX}(\tau) = 16 + \frac{9}{1+6\tau^2}$.

6. Consider a Markov chain with state {0, 1, 2} and transition probability matrix $P = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{pmatrix}$. Draw the state transition diagram.

7. Define Markovian Queueing Models.

8. Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes.

(a) What is the average number of customers in the system?

(b) What is the average time of a customer spends in the system?

9. What do you mean by a series queue with blocking?

10. Define 'Bottle neck' of the system in queue networks.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Ten percent of the tools produced in a certain manufacturing company turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective by using

(1) binomial distribution

(2) The Poisson approximation to the binomial distribution. (8)

(ii) The number of typing mistakes that a typist makes on a given page has a Poisson distribution with a mean of 3 mistakes. What is the probability that she makes

(1) Fewer than 4 mistakes

(2) No mistakes on a given page. (8)

Or

(b) (i) The lifetime X of particular brand of batteries is exponentially distributed with a mean of 4 weeks. Determine

(1) The mean and variance of X.

(2) What is the probability that the battery life exceeds 2 weeks?

(3) Given that the battery has lasted 6 weeks, what is the probability that it will last at least another 5 weeks? (8)

(ii) Find the moment generating function of Geometric random variable and hence determine the mean and variance. (8)

12. (a) Obtain the equations of the lines of regression from the following data: (16)

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

Or

(b) (i) The joint pdf of random variable X and Y is given by

$$f(x, y) = \begin{cases} k(4-x-y), & 0<x<2,\ 0<y<2 \\ 0; & \text{otherwise.} \end{cases}$$

(1) Determine the value of k.

(2) Find the marginal probability density function of X and Y. (8)

(ii) Let $X_1, X_2, \ldots, X_{100}$ be independent identically distributed random variables with $\mu = 2$ and $\sigma^2 = \frac{1}{4}$. Find $P(192 < X_1 + X_2 + \ldots + X_{100} < 210)$. (8)

13. (a) (i) Two random processes $X(t)$ and $Y(t)$ are defined by $X(t) = A\cos\lambda t + B\sin\lambda t$ and $Y(t) = B\cos\lambda t - A\sin\lambda t$. Show that $X(t)$ and $Y(t)$ are jointly wide sense stationary, if A and B are uncorrelated random variables with zero means and the same variances and λ is a constant. (10)

(ii) Prove that difference of two independent Poisson processes is not a Poisson process. (6)

Or

(b) (i) A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X(n)\}$. Also find $P\{X_2 = 6\}$. (8)

(ii) The three – state Markov chain is given by the transition probability matrix $P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$. Prove that the chain is irreducible and all the states are aperiodic and non null persistent. (8)

14. (a) (i) A T.V. repairman finds that the time spend on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of average set just brought? (8)

(ii) Obtain the steady state probabilities for a multi server queuing model with infinite capacity and hence obtain an expression for L_q. (8)

Or

(b) (i) A telephone exchange has two long distance operators. It is observed that, during the peak load long distance calls arrive in a Poisson flashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes.

Find

(1) The probability a subscriber will have to wait for long distance call during the peak hours of the day.

(2) If the subscribers will wait and are serviced in turn, what is the expected waiting time? (8)