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Reg. No. :

Question Paper Code : 53254

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third/Fourth/Sixth/Eighth Semester

Environmental Engineering

MA 6468 – PROBABILITY AND STATISTICS

(Common to Mechanical Engineering (Sandwich), Agriculture Engineering, Industrial Engineering, Industrial Engineering and Management, Manufacturing Engineering, Bio Technology, Chemical Engineering, Fashion Technology, Food Technology, Handloom and Textile Technology, Petrochemical Technology, Petroleum Engineering, Pharmaceutical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

(Use of statistical tables are permitted in the exam hall)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The probability mass function of a R.V. X is $P(X=x)=\binom{2}{x}\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{2-x}$, $x=0, 1, 2$. Compute $P(X<2)$.

2. A R.V X is uniformly distributed on $(-5, 15)$. Compute $E\left(e^{-\frac{1}{5}X}\right)$.

3. The joint p.d.f of a R.V. (X, Y) is given by $f(x, y)=\begin{cases} e^{-(x+y)}, & x\geq 0,\ y\geq 0 \\ 0 & \text{, otherwise} \end{cases}$

   Calculate $P(X<Y)$.

4. State the central limit theorem for independent and identically distributed random variables.

=1.94, $s = 0.10$ and $n = 100$, test the null hypothesis $H_0 : \mu = 2$ against lternative hypothesis $H_1 : \mu \neq 2$ at $\alpha = 0.01$ level of significance.

e (a) critical region (b) degrees of freedom.

the identity for total sum of squares for one-way of analysis of variance.

2×2 Latin square possible? Explain.

e the formulae for control values (central line, UCL and LCL) of a c-chart.

rol chart for $\overline{X}$ and R are to be set for an important quantity acteristic. The sample size is $n = 2$, $\overline{x}_i$'s and r_i's are computed for each of reliminary samples. The sample data are $\sum_{i=1}^{35} \overline{x}_i = 7805$ and $\sum_{i=1}^{35} r_i = 1200$. the trail control limits for $\overline{X}$ and R.

PART B — (5 × 16 = 80 marks)

(i) Suppose that R.V X has the probability mass function $P(X = x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, 3, \ldots \\ 0, & otherwise \end{cases}$

Find (1) $P(X > 3)$ (2) $P(X > 5/X > 3)$ (3) moment generating function of R.V. X and hence obtain $E(X)$. (8)

(ii) The p.d.f of a R.V. X is given by $f(x) = \begin{cases} Kx(2-x), & 0 < x < 2 \\ 0 & , otherwise \end{cases}$

(1) Find the value of 'K'

(2) Obtain Cumulative distribution function $f(x)$ of a R.V. X and hence compute $P(1 < X < 3/2)$

(3) Calculate $E(X)$ also. (8)

Or

(i) Identify the distribution of the R.V X with the following moment generating function $M_X(t) = e^{3(e^t - 1)}$. Hence obtain $E(X)$, $\text{var}(X)$ and $P(X = 5)$. (8)

(ii) If a R.V X has the p.d.f $f(x) = \begin{cases} xe^{-x}, & x > 0 \\ 0, & x \le 0 \end{cases}$, Obtain (1) the moment generating function $M_X(t)$ of R.V. X (2) $E(X)$ (3) cumulative distribution function, $F(x)$ of the R.V. X (4) $P(X < 2)$.

12. (a) (i) Let the joint probability mass function of a R.V. (X, Y) be given as

$$P(X = x, Y = y) = \begin{cases} \dfrac{x+y}{12}, & x = 1, 2, \ y = 1, 2 \\ 0 & , otherwise \end{cases}$$

(1) Determine the marginal probability mass functions of X and Y

(2) Find the conditional probability mass function $P\{X = x | Y = 2\}$, $x = 1, 2$.

(3) Are of R.V.s X and Y independent? Explain. (8)

(ii) Suppose the R.V (X, Y) has the joint p.d.f.

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \ y > 0 \\ 0 & , otherwise \end{cases}$$

Find the joint p.d.f. of R.Vs $U = \dfrac{X}{Y}$ and $V = Y$ and hence obtain the marginal p.d.f. of the R.V. U. (8)

Or

(b) (i) The joint probability mass function of two R.Vs X and Y is given as $P(X = 0, Y = 0) = 1/2$, $P(X = 1, Y = 0) = 1/8$, $P(X = 0, Y = 1) = 1/8$ and $P(X = 1, Y = 1) = 1/4$. Compute (1) Covariance, $\text{cov}(X, Y)$ of R.Vs X and Y (2) $P(X + Y \le 1)$. (8)

(ii) Given the joint p.d.f of R.V (X, Y) as

$$f(x, y) = \begin{cases} \frac{1}{4}(1 + xy), & |x| < 1, \ |y| < 1 \\ 0 & , otherwise \end{cases}$$

Find,

(1) the marginal p.d.fs of X and Y

(2) the conditional p.d.f $f(x|y)$ of X given $Y = y$ and the conditional p.d.f. $w(y|x)$ of Y given $X = x$. (8)

(i) From the previous experience, we have obtained the following data for two samples from two populations.

Sample 1 : $n_1 = 7$ $\quad \bar{x}_1 = 81$ $\quad s_1 = 8$

Sample 2 : $n_2 = 10$ $\quad \bar{x}_2 = 72$ $\quad s_2 = 6$

Assume that the population variances are equal but unknown. Test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ at $\alpha = 0.05$ level of significance. (8)

(ii) Let X be the number of errors found in a total of $n = 85$ software products, Use the goodness of fit statistics for the following data to test the hypothesis at $\alpha = 0.05$ significance level that the distribution X is Poisson with a mean of $\lambda = 3$. (8)

No. of Errors :	0	1	2	3	4	5	6	7	8
Frequency :	3	14	20	25	14	6	2	0	1

Or

(i) Two scientists Dr. X and Dr. Y find a previously unknown type fish in a remote river. They both trap some fish from different location some distance apart. The weight of their fish are as follows (in kg) :

Dr. X :	0.15	0.18	0.25	0.36	0.42	0.44	–	–
Dr. Y :	0.25	0.26	0.26	0.30	0.32	0.33	0.37	0.37

Test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 = \sigma_1^2 \neq \sigma_1^2$ at $\alpha = 0.1$ significance level. (8)

(ii) Tests of the fidelity and selectivity of 190 radio receivers produced the results shown in the following contingency table :

		Fidelity		
		Low	Average	High
Selectivity	Low	6	12	32
	Average	33	61	18
	High	13	15	0

Use $\alpha = 0.01$ level of significance to test whether there is a relationship (dependence) between fidelity and selectivity. (8)

14. (a) An experiment was designed to study the performance of 4 different detergents for cleaning fuel injectors. The following "Cleanness" readings were obtained with specifically designed equipment for 12 tanks of gas distributed over 3 different models of engines :

		Engine		
		I	II	III
Detergent	A	45	43	51
	B	47	46	52
	C	48	50	55
	D	42	37	49

Looking at the detergents as treatments and the engines as blocks, obtain the appropriate two-way analysis of variance table and test at α = 0.01 level of significance whether there are differences in the detergents or in the engines. (16)

Or

(b) A former wishes to test the effects of four different fertilizers A, B, C, D on the yield of wheat. In order to eliminate sources of error due to variability in soil fatality , he uses the fertilizers in a Latin square arrangement as indicated in the following table, where the numbers indicate yields in bushes per unit area :

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

Perform an analysis of variance to determine if there is a significant difference between the fertilizers at $\alpha = 0.05$ significance level. (16)

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en inspection sample lots of five amplifiers each are drawn from roduction. The following table lists the average life and range of the ower output obtained for each amplifier. Construct $\overline{X}$–chart and –chart. Comment on the state of control. (16)

No :	1	2	3	4	5	6	7	8	9	10
	11.0	12.0	12.8	14.0	13.6	12.8	11.8	12.9	13.0	11.8
	4	4	6	4	6	5	5	6	4	6

Or

hirty-five successive samples of 100 casting each taken from a roduction line contained, respectively,

, 3, 5, 3, 5, 0, 3, 2, 3, 5, 6, 5, 9, 1, 2, 4, 5, 2, 0, 10, 3, 6, 3, 2, 5, 6, 3, 3, 2, 5, , 0, 7, 4 and 3 defectives. If the fraction defective is to be maintained at = 0.02. Construct a p-chart for these data and state whether or not the andard is being met. (16)

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