(b) (i) State and prove Wiener-Khintchine theorem. (8)

(ii) If the cross power spectral density of $X(t)$ and $Y(t)$ is $S_{XY}(w)=\begin{cases} a+\dfrac{ibw}{\alpha}; -\alpha < w < \alpha, \alpha > 0 \\ 0, \quad \text{otherwise} \end{cases}$ where a and b are constants. Find the cross correlation function. (8)

15. (a) (i) Prove that if the input to a time-invariant stable linear system is a wide sense process then the output also is a wide sense process. (8)

(ii) A random process $X(t)$ with $R_{XX}(\tau)=e^{-2|\tau|}$ is the input to a linear system whose impulse response is $h(t)=2e^{-t}, t>0$. Find the cross correlation coefficient $R_{XY}(\tau)$ between the input process $X(t)$ and output process $Y(t)$. (8)

Or

(b) (i) Let $X(t)$ be a wide sense stationary process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$. Prove that $S_{YY}(w)=|H(w)|^2 S_{XX}(w)$ where $H(w)$ is the Fourier transform of $h(t)$. (8)

(ii) Let $Y(t)=X(t)+N(t)$ be a wide sense stationary process where $X(t)$ is the actual signal and $N(t)$ is the zero mean noise process with variance σ_N^2, and independent of $X(t)$. Find the power spectral density of $Y(t)$. (8)

———————

29/4/19

Reg. No. :

Question Paper Code : 31265

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Electronics and Communication Engineering

MA 2261 — PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The moment generating function of a random variable X is given by $M(t)=e^{3(e^t-1)}$. What is $P[X=0]$?

2. An experiment succeeds twice as often as it fails. Find the chance that in the next 4 trials, there shall be at least one success.

3. Find the marginal density functions of X and Y if

$$f(x,y)=\begin{cases} \frac{6}{5}\left(x+y^2\right), 0 \le x \le 1, 0 \le y \le 1 \\ 0 \ \ \textit{otherwise} \end{cases}$$

4. Find the acute angle between the two lines of regression, assuming the two lines of regression.

5. Define wide sense stationary process.

6. Show that a binomial process is Markov.

7. Define power spectral density function.

8. State Wiener-Khinchine theorem.

9. Define a linear system with random output.

10. State any two properties of cross power density spectrum.

www.Vidyarthiplus.com

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the probability density of X is give by $f(x)=\begin{cases}2(1-x) \text{ for } 0<x<1\\ 0, \text{ otherwise}\end{cases}$ find its r^{th} moment. Hence, evaluate $E[(2X+1)^2]$. (6)

(ii) Find MGF corresponding to the distribution $f(\theta)=\begin{cases}\frac{1}{2}e^{-\theta/2}, \theta>0\\ 0, \text{ otherwise}\end{cases}$ and hence find its mean and variance. (6)

(iii) Show that for the Probability function $p(x)=P(X-x)=\begin{cases}\frac{1}{x(x+1)}, x=1,2,3...\\ 0, \text{ otherwise}\end{cases}$ $E(X)$ does not exist. (4)

Or

(b) (i) Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable X normally distributed with mean 37.6 cc/mm and S.D 4.6 cc/min Determine the probability that during a period of T.M. a person's Oxygen consumption will be reduced by

(1) at least 44.5 cc/min

(2) utmost 35.0 cc/min

(3) anywhere from 30.0 to 40.0 cc/mm. (8)

(ii) The random variable X has exponential distribution with

$f(x)=f(X)=f(x)=\begin{cases}e^{-x}, & 0<x<\infty\\ 0, \text{ otherwise.}\end{cases}$

Find the density function of the variable given by

(1) $Y=3X+5$

(2) $Y=X^2$. (8)

12. (a) (i) The joint PMF of two random variables X and Y is given by $P_{XY}(x,y)=\begin{cases}K(2x+y), & x=1,2; y=1,2\\ 0, \text{ otherwise}\end{cases}$, where K is constant

(1) Find K

(2) Find the marginal PMFs of X and Y. (8)

(ii) Assume that the random variable S_n is the sum of 48 independent experimental values of the random variable X whose PDF is given by $f_X(x)=\begin{cases}\frac{1}{3}, & 1\le x\le 4\\ 0, \text{ otherwise}\end{cases}$. Find the probability that S_n lies in the range $108\le S_n\le 126$. (8)

Or

(b) (i) Two random variables X and Y are related as $Y=4X+9$. Find the correlation coefficient between X and Y. (8)

(ii) If the density function is defined by $f(x,y)=\frac{y}{(1+x)^4}e^{\frac{-y}{1+x}}$, $x\ge 0, y\ge 0$ then obtain the regression equation of Y on X for the distribution. (8)

13. (a) (i) If the two RVs A_r and B_r are uncorrelated with zero mean and $E(A_r^2)=E(B_r^2)=\sigma_r^2$, show that the process.

$x(t)=\sum_{r=1}^{n}(A_r\cos w_r t+B_r\sin w_r t)$ is wide-sense stationary. (8)

(ii) If $\{x(t)\}$ is a Gaussian process with $\mu(t)=10$ and $C(t_1,t_2)=16e^{-|t_1-t_2|}$, find the probability that

(1) $X(10)\le 8$ and

(2) $|X(10)-X(6)|\le 4$. (8)

Or

(b) (i) Define Random telegraph signal process and prove that it is wide-sense stationary. (8)

(ii) Prove that sum of two independent Poisson processes is a Poisson process. (8)

14. (a) (i) Define spectral density of a stationary random process $X(t)$ Prove that for a real random process $X(t)$ the power spectral density is an even function. (8)

(ii) Two random processes $X(t)$ and $Y(t)$ are defined as follows :

$X(t)=A\cos(\omega t+\theta)$ and $Y(t)=B\sin(wt+\theta)$ where A, B and w are constants ; θ is a uniform random variable over $(0,2\pi)$. Find the cross correlation function of $X(t)$ and $Y(t)$. (8)

Or